QUANTITATIVE INTERPRETATION OF ENERGY-BASED SYSTEMS AND INDEX OF THEIR RELIABILITY

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Abstract

The paper presents a proposal of quantitative interpretation for operation of energy-based systems (e.g. diesel engines, gas turbine engines, steam turbines, steam and water boilers), which (just like in physics: the operations of Hamilton and Maupertius and the operation resulting from changes of body momentum) is considered as a physical quantity with a joule-second [joule × second; Js] as a unit of measure. It has been also showed that interpretation of operation can be considered as a reliability index and in special cases – as a safety index of such energy-based systems’ operation. To give the grounds for such usability of the mentioned energy-based systems’ operation the homogeneous process of Poisson has been applied. This process enabled constructing a model of run of getting worse (decreasing) operation of a gas turbine engine with the lapse of operation time. Thus, such model is a random process of homogeneous and independent gains in drops of energy generated by the engine, as the result of using it in determined time.

1. Introduction

Ensuring safe operation of an energy-based system (e.g. installed inside a ship: diesel engines, gas turbine engines, steam turbines, steam and water boilers, etc.) needs having an adequate quantity of energy being obtained from chemical energy contained in fuel consumed by engines. Action which ensures delivering the wanted quantity of energy in proper time strictly depends on taken decisions.

Taking decisions in time of operation of energy-based systems (e.g. installed inside sea-going ships) is always realized in a stochastic decision-making situation, thus in the conditions of uncertainty (in conditions of statistic risk). That means, that there is a need of using the rules of the calculus of probability and the inductive (mathematical) statistics. Operating decisions are taken at the very beginning (before starting) and during operation of the ship. That means that these decisions are taken at least once, on the basis of primary information (obtained e.g. during reliability tests of energy-based systems and their particular elements, or from the bank of information concerning similar systems) which can be named a priori, and next on the basis of information obtained during operation of the systems (e.g. as the result of application of diagnostics, not only technical one), which can be named a posteriori information.
Decisions, taken at the beginning of operation, are indispensable to plan the process of using and operating the systems. These decisions have to take the risk into account, of which the estimation is a probability of taking a wrong decision, resulting from [1, 5]:

- impossibility of precise estimation of unknown parameters of distributions of random variables, which are the states of the process of operating the energy-based systems and their particular elements;
- lack of possibility to elaborate entirely or/and enough reliable information demanded to take the right decision.

The first case generates random mistakes of which the estimation is called the stochastic precision of inferring and the second one – random mistakes and such mistakes which can be considered as not random (systematic). Establishing the last kind of mistakes is a problem, that I suggest to name a problem of accuracy or precision of inferring. Determination of these mistakes together is a problem of statistic precision of inferring. The accuracy of inferring results from the current level of scientific and useful knowledge, and the precision of inferring results from not appreciating some information and leaving it out of account, although he/she could make himself/herself sure if it was really unimportant. However, in time of operating energy-based systems the reasons of taking wrong or irrational decisions are difficulties in establishing a completed and sufficiently reliable diagnosis on the technical state of the engines as well as a similar diagnosis referring to the expected outer conditions (of weather and sea) which can appear during operation [1, 4, 12, 16].

In the presented above situation for making decisions, making the rational decision is possible in case of applying the statistic theory of making decisions and thereby the expected value of consequences as the criterion for making such decision [5, 7]. Determining a set of decisions which if necessary, may be taken in agreement with the taken criterion of optimisation, needs identification of the problem of safety and reliability of the energy-based system in respect to its operation.

2. General identification of safety problem for energy-based systems

In order to avoid any threat for energy-based systems, man has to make proper decisions and take actions resulting from the decisions, both in the phase of preparing the system to perform a given task and during the course of task performance, and after the task was finished. This action, of course, has to be efficient, so – purposeful, energetic and economical [6-9]. Such action, just like another one, demands using proper quantity of energy in determined time. Thus, it can be considered as a physical value expressed with the unit of measure [joule \times \text{second}; \text{Js}] and interpreted (generally, in a deterministic formulation) in the form of dependence:

\[ A = Ut \]  \hspace{1cm} (1)

where: \( A \) – action, \( U \) – energy used in the action \( A \), \( t \) – time of energy \( U \) use.

Interpretation of action, presented by the dependence (1), has its equivalent in physics (quantum mechanics) - Planck constant (\( h \)) because [2, 10, 15]:

\[ E_\nu = h \nu \rightarrow h = E_\nu \nu^{-1} \]  \hspace{1cm} (2)

where: \( E_\nu \) – energy of one quantum of electromagnetic radiation, \( \nu \) – frequency of energy quantization, \( h \) – Planck constant of which the unit of measure [joule \times \text{second}; \text{Js}] has been called action.
Action expressed by the formula (1) has also its equivalents in thermodynamics and tribology. In thermodynamics there may be considered two methods of changing energy in time $t$: work $L$ and heat $Q$ [11, 14]. The operation of systems can be expressed by formulas:

$$A_L = Lt, \ \ A_Q = Qt$$ (3)

In tribology the operation of tribological systems can be interpreted in a similar way considering the work of friction ($W_T$) [16], done in time $t$. Then, the work can be expressed as follows:

$$A_T = W_T t$$ (4)

Such understood work has to be of course, comprehensively analysed. In order to do that, proper indexes which determine efficiency of work, e.g. universal efficiency meters [8], are needed.

Action which results from tending to keep the safety of a energy-based system, in agreement with the dependence (1), can be:

- Demanded action ($A_W$) in the situation (in which the system find itself), so such action that ensures (enables) keeping the safety of the energy-based system;
- possible action ($A_M$) in the situation (in which the system find itself), so such action that may be, but doesn’t have to be, sufficient to ensure (keep) the safety of an energy-based system.

In accordance with the dependence (1), the actions can be expressed by the dependencies:

$$A_W = U_W t_W$$ (5)

$$A_M = U_M t_M$$ (6)

Safe operation of a system is possible only when the possible action ($A_M$) amounts at least to the demanded action ($A_W$), so the action indispensable to keep the safety in the situation in which the operated system finds itself. That means that safe operation of a system is possible, if:

$$A_M \geq A_W$$ (7)

Thus, any threat for a system occurs when:

$$A_M < A_W$$ (8)

We can predict that action for the bigger than the wanted scale, is less efficient. Thus, the need is to tend to $A_M = A_W$.

From the considerations results that the following cases of threat for an energy-based system may be taken into consideration:

1) $t_M < t_W$, if simultaneously $U_M = U_W$;
2) $U_M < U_W$, if simultaneously $t_M = t_W$;
3) $U_M < U_W$, if simultaneously $t_M < t_W$;
4) $U_M > U_W$ if simultaneously $t_M < t_W$, but in consequence $A_M < A_W$;
5) $U_M < U_W$, if simultaneously $t_M > t_W$, but in consequence $A_M < A_W$.

The mentioned conditions can be interpreted as follows: the first of them reflects the situation when the energy-based system’s operator has not time needed to ensure the safe operation of the energy-based system. The second condition reflects a situation when although having the needed time the safety of the energy-based system cannot be ensured because of lack of the proper quantity of energy. The third condition reflects the most difficult situation in which a gas turbine engine may find itself because of not only insufficient energy but also the lack of time to ensure safe operation to the energy-based system. The fourth condition reflects such situation when there is excess energy comparing with the needed quantity to ensure safe operation to the energy-based system but the time in which the energy should be made out is insufficient. The effects of this situation may be similar to the mentioned ones in the case of the first condition. The last condition reflects such situation when the needed energy to ensure the safe operation to the energy-based system is insufficient and cannot be increased although the time of action in which this energy could be increased before a threat to the energy-based system occurs, is long. This situation may be caused by extensive damages in constructional structure (damages called break-downs) of the energy-based system, being of the essential meaning for its safety.

Presented considerations concerning quantitative formulation of action to ensure the safety of a energy-based system, can be (and should be) developed by applying the theory of stochastic processes. This follows from that generating energy by a energy-based system in time of operating is a random process. This process in conditions of fixed states of the energy-based systems’ operation, can be a set of random variables $U_t$ of not large (and that’s why unimportant from the practical point of view) variation. However, in the reality changes of energy in time of operating can be (and should be) considered as a stochastic process $\{U(t); t \geq 0\}$ with a defined expected value $E[U(t)]$ and variation $V^2[U(t)]$.

Examination of the process in any interval $(t_0, t_0 + t)$ demands considering its momentary states (for each time $t$) which are random variables $U_t$ with expected values $E(U_t)$ and variation $V^2(U_t)$, dependent on the value $t$. It is obvious that both: the expected value and variation of the process $\{U(t); t \geq 0\}$ depend on time $t$ because for its different values $E(U_t)$ and also $V^2(U_t)$ can be different. But $E[U(t)]$ and $V^2[U(t)]$ are not random functions because $E(U_t)$ and also $V^2(U_t)$ are constant quantities for a given value $t$ and defined set of random variables values $U$, and they are not random variables. Thus, the dependence (1) can be presented as follows:

$$A(t) = \int_0^t E[U(\tau)]d\tau \quad (9)$$

Considering the safety, it may be important to take more careful decisions or more risky ones, thus it is necessary to do estimation not only in a point but also in an interval, so in the formula (9) instead of $E[U(t)]$ it should be put the value of the bottom limit of the confidence interval $E_d[U(t)]$ if the decision should be more careful or the value from the top limit of the confidence interval $E_g[U(t)]$ if the risky decision is admissible. It is obvious that when in the particular intervals $\Delta t_i$ of time $t$ (of generating energy) the expected values $E(U_t)$ can be considered as constant, the mark of integral should be replaced in the formula (9) by the symbol of sum.

In a concrete use of the interpretation of action, according to the dependence (1) or (9), the operation of an energy-based system and the operation of its subsystems can be expressed in the form of different formulas according to:
• type of a subsystem which in defined time generates the energy to meet the need of the whole system for operation;
• class of stochastic processes of which changes of energy consumed during the course of the system operation, may be included to.

Taking into consideration the presented above interpretation of the energy-based systems’ operation it is possible to define a reliability state in which the system finds itself.

3. Interpretation of reliability states for an energy-based system

Taking into account the dependencies (7) and (8) it may be accepted that each energy-based system is in the state of ability (and is able to perform a task) if meets the dependence (7). Otherwise, in case of inequality (8) it should be accepted that the energy-based system is in the state of disability. That means that such an energy-based system should be considered as a failed one although the energy is still transformed in it. The dependence (7) is satisfied if such a system like a gas turbine engine may be loaded according to the external characteristic of maximal power in the time interval suggested by a producer. In case when the engine may not be loaded (without any threat of failure) the dependence (7) can be satisfied only if in time of performing the task it does not occur the need to load the engine according to this characteristic. Otherwise, the dependence (7) is not satisfied and the engine (as mentioned) should be considered as damaged.

Inferring about usability of particular energy-based system for realization of exactly defined tasks can be made after comparing fields of system’s action: the demanded one $A_w$ and the possible one $A_M$. From the presented above considerations results that the system operation in this formulation means:

• testing changes of the demanded work $L_{eW}$, to be done by the energy-based system in the demanded time $t_W$, so in time in which a transport task should be finished;
• testing changes of the possible work $L_{eM}$, to be done by the energy-based system in possible time $t_M$, so in time in which the system can be correctly operated.

Considering energy-based system’s operation as a measure of (full or partial) ability of an energy-based system to perform the task, demands first of all defining the classes of model states among which its technical state could be classified. According to the dependence (7) the energy-based system finds itself in the state of ability (so in the state which makes performing tasks possible), if:

\[
\begin{align*}
1) & \ t_M \geq t_M \text{ if at the same time } L_{eM} \geq L_{eW} \\
2) & \ t_M = t_M \text{ if at the same time } L_{eM} = L_{eW} \\
3) & \ t_M \geq t_M \text{ if at the same time } L_{eM} = L_{eW} \\
4) & \ t_M = t_M \text{ if at the same time } L_{eM} \geq L_{eW}.
\end{align*}
\]

(10)

In case when none of dependencies (10) can be satisfied the energy-based system should be considered as disable to perform tasks although it is able to convert chemical energy into mechanical energy which enables performing the work $L_e$ by it [3, 13]. Thus, operation $A_e$ of any energy-based system, being analyzed with regard to the dependence (10), can be accepted as a factor of its reliability. In case when the dependencies (10) are not satisfied, so the inequality (8) takes place, the operation $A_M$ of the given energy-based system can also be the measure (factor) of safe operation. Of course, the factors of reliability and safety for the energy-based systems’ operation may also be the generally known factors referring to the systems’ operation in the version suggested here in this paper. In this case, as the reliability...
measures can be considered the probabilities of satisfying the equations (10), being the probabilities of correct operation of the energy-based system and its performance of the demanded task. For elaboration of these reliability factors (and in case of taking sea accidents into account – safety factors) the homogeneous process of Poison can be applied as the model of the process of decreasing mechanical energy (so also the work \( L_e \)) as the result of wear of the energy-based system \([6, 7, 9]\). Applying this process, the following physical interpretation of the process of decreasing work \( L_e \) by a constant value \( e \) can be expressed: from the moment of starting operation of a energy-based system (it can be the moment \( t_0 = 0 \)) to the moment of recorded for the first time by a measuring device, the event \( E \) which is a decrease (as the result of wear of the system) of work \( L_e \) by the value \( \Delta L_e = e \), it can be performed any value of work \( L_e \) (including the maximal one) in particular time intervals of energy-based system’s operation. Further use of the energy-based system causes occurring next drops of the values of work \( L_e \), by the next homogeneous values \( e \), recorded by a measuring device. Therefore, in case of recording the cumulated quantity \( B_t \) of occurred events \( E \) up to the moment \( t \) described by the homogeneous process of Poison, the total decline of work \( L_e \) by the value \( L_e(t) \) to the moment \( t \) can be presented by the dependence:

\[
L_e(t) = eB_t
\]

where: \( e \) – quantum of energy, \( B_t \) – cumulated number of events \( E \) appeared (recorded) up to the moment \( t \).

at which the random variable \( B_t \) is (as it’s known) of the distribution \([1, 6]\)

\[
P(B_t = k) = \frac{(\lambda t)^k}{k!}\exp(-\lambda t); \quad k = 1, 2, ..., \quad (12)
\]

where: \( \lambda \) – constant value (\( \lambda = \text{idem} \)) interpreted as the intensity of decreasing work \( L_e \) by the same values \( e \), recorded in time of the research; \( \lambda > 0 \).

The expected value and the variation of the process of growing the quantity of events \( E \), so decreasing the work \( L_e \) by values \( e \), recorded in turn, can be presented as follows:

\[
E(B_t) = \lambda t; \quad D^2(B_t) = \lambda t \quad (13)
\]

Thus, according to the dependence (11) and formulas (13) the expected value and the standard deviation of decreasing work \( L_e \) performed by the energy-based system up to the moment \( t \), can be expressed by the formulas \([6, 7]\):

\[
E[\Delta L_e(t)] = eE(B_t) = e\lambda t \quad \sigma_L(t) = e\sqrt{D^2(B_t)} = e\sqrt{\lambda t} \quad (14)
\]

Taking into account the fact that a brand new energy-based system can (when \( t = 0 \)) perform the biggest work, so \( L_e(0) = L_{\text{max}} \), the mathematical dependence describing the decline of this work with the lapse of time, can be expressed by the formula \([6, 7]\):
From the formula (15) results that for any moment $t$ the work $L_e$ which can be performed by the system, can be determined and from the formula (12) – that it can be determined the probability of occurring such the decline of work $L_e$ as the result of wear of the energy-based system, what makes performing the task impossible. Thus, the probability $P(B_t = k; \ k = 1, 2, \ldots, n)$ determined by the formula (12), can be considered as a reliability factor of the energy-based system. The probability can be also a safety factor of energy-based system’s operation in case if it concerns such the decline of work $L_e$ which may lead to an accident.

4. Summary

Operation of any energy-based system has been presented as a measure (index) of its reliability and safety. In the presented suggestion the operation is understood as generating and processing the energy by a technical system in determined time, which enables the energy-based system to perform the useful (effective) work $L_e$. The operation has been considered as a physical quantity which can be expressed (just like in physics – e.g. Hamilton’s operation, Maupertius operation and the others [6, 7, 9]) with a number and the unit of measure: a joule-second [joule × second; Js].

Graphic interpretation of the dependence (15) is presented in Fig. 1.

\[
L_e(t) = \begin{cases} 
L_{\text{emax}} & \text{dla } t = 0 \\
L_{\text{emax}} - e\lambda t \pm e\sqrt{\lambda t} & \text{dla } t > 0 
\end{cases}
\]
References


