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IMPACT OF THE METHOD OF ELABORATION OF A LOAD SPECTRUM ON THE RESULTS OF THE CALCULATION OF S355J0 STEEL FATIGUE LIFE

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Abstract

The evaluation of fatigue life of the elements of machines which have been subjected to service load can be carried out through empirical examination or calculations. The calculations of fatigue life require the knowledge of fatigue characteristics, assuming the Palgrem-Miner rule and elaborating on the spectrum of the service load. The way of elaboration on the load spectrum is connected among other things with taking the appropriate normalization method into account. In this work the analysis of the influence of selected methods of normalization on the calculated fatigue life has been depicted. In the analysis the following methods have been applied: counting of local extrema, counting the branch span, pairs of spans and full cycles. The calculation has been done for exploitation loads run which was a result of measurements.

***Keywords**: fatigue life, random service load, normalization methods*

Nomenclature

A_5 – elongation [%],
$D_{()}$ – value of the statistical parameter for the course in discrete form,
$L_{()}$ – value of the statistical parameter for the course in local extrema form,
N – cycle number – general notation (fatigue life),
N_e – number of avarage value crossings through increasing and decreasing semicycles,
N_{ij} – number of cycles to fatigue crack for values S_{aij} and S_{mij},
N_0 – cycle number – fatigue life corresponding with fatigue limit,
$R = S_{min}/ S_{max}$ – cycle asymmetry ratio,
R_e – material yield point [MPa],
R_m – material tensile strength [MPa],
R_{-1} – fatigue limit under oscillating load (R = -1) for N_0 cycle number, [MPa],
S – specimen stress – general notation, [MPa],
$S_a = 0,5(S_{max} - S_{min})$ – sinusoidal cycle stress amplitude [MPa],
$S_m = 0,5(S_{max} + S_{min})$ – mean sinusoidal cycle stress [MPa],
S_{max} – maximum sinusoidal cycle stress [MPa],
S_{min} – minimum sinusoidal cycle stress [MPa],
Z – contraction [%],

m_0 – exponent in formula describing Wöhler fatigue diagram for oscillating load (R = -1),
n_{ij} – number of cycles with variable values Saij and Smij,

1. Introduction

Calculating of fatigue life of structural components which have been subjected in service conditions to random load concern three problems: description of fatigue properties of a structural component, Palgrem-Miner rule and elaborating the model of the service load.

The fatigue properties of elements can be described by relations in system $N(S_a)$ or $N(S_m, S_a)$. Assuming that cycles of values S_m and S_a from the wide range of inequalities belong to the composition of service load, more beneficial is (for the sake of calculation accuracy) application of two-parametric model of fatigue properties of element (two-parametric fatigue characteristics). The problems mentioned above have been described in details in references [7, 8].

Another element necessary to do the calculations is taking Palgrem-Miner rule as basis. The detailed characteristics of the methods has been described in the reference [3].

The third problem concerns the method of elaboration of the random load spectrum. The first stage in process of elaboration of the spectrum mentioned above is so called normalization, the main task of which is to substitute the course of loading being a record of subsequent extreme values (e.g. stresses) over time on set of sinusoidal cycles described with parameters S_m and S_a or S_{min} and S_{max}. The determined set of data enables us to elaborate on load spectrum, which may be: statistical function, block load spectrum or correlation table. Several methods of normalization are known, among which the most often applied are: peak counting method, range counting method, range pair count method, full cycles count method and rainflow count method. The detailed characteristics of the methods mentioned above has been described in references [2, 4, 5, 6]. On their basis it can be assumed that in the case of elaborating on random courses of a narrow spectrum the choice of the normalization method does not impact significantly on the calculated fatigue life. However, in case of courses of loads of a wide spectrum the methods leading to determination of the full sinusoidal cycles are recommended. To these methods belong: range pair count method, full cycles count method and rainflow count method.

The aim of this work is to compare the results of fatigue life received from calculations for load spectrums elaborated with use of various methods of normalization of the chosen operating course.

The range of this work includes presentation of service load, statistical evaluation normalization of loads and analysis of results of fatigue life calculations received for different methods of normalization.

2. Exploitation loads run

For the research purposes the course of stresses changes of a car steering spindle registered while driving forwards with speed of 30km/h on the straight road without elevations, on the concrete pavement (fig.1a) has been assumed. The registered changes of stresses have been depicted in relative values related to the maximum value present in the course. After that the course of loading has been divided into 20 ranges and the local extrema have been determined (fig. 1b). Figure1c and 1d depict short sections of courses in discrete form and form of local extrema, on which the values belonging to them have been marked with points. The course in form of local extrema is the simplified form of course registered during measurements. Its simplification is connected to determination of local extrema i.e. minima and maxima present in turns, situated in various class ranges.

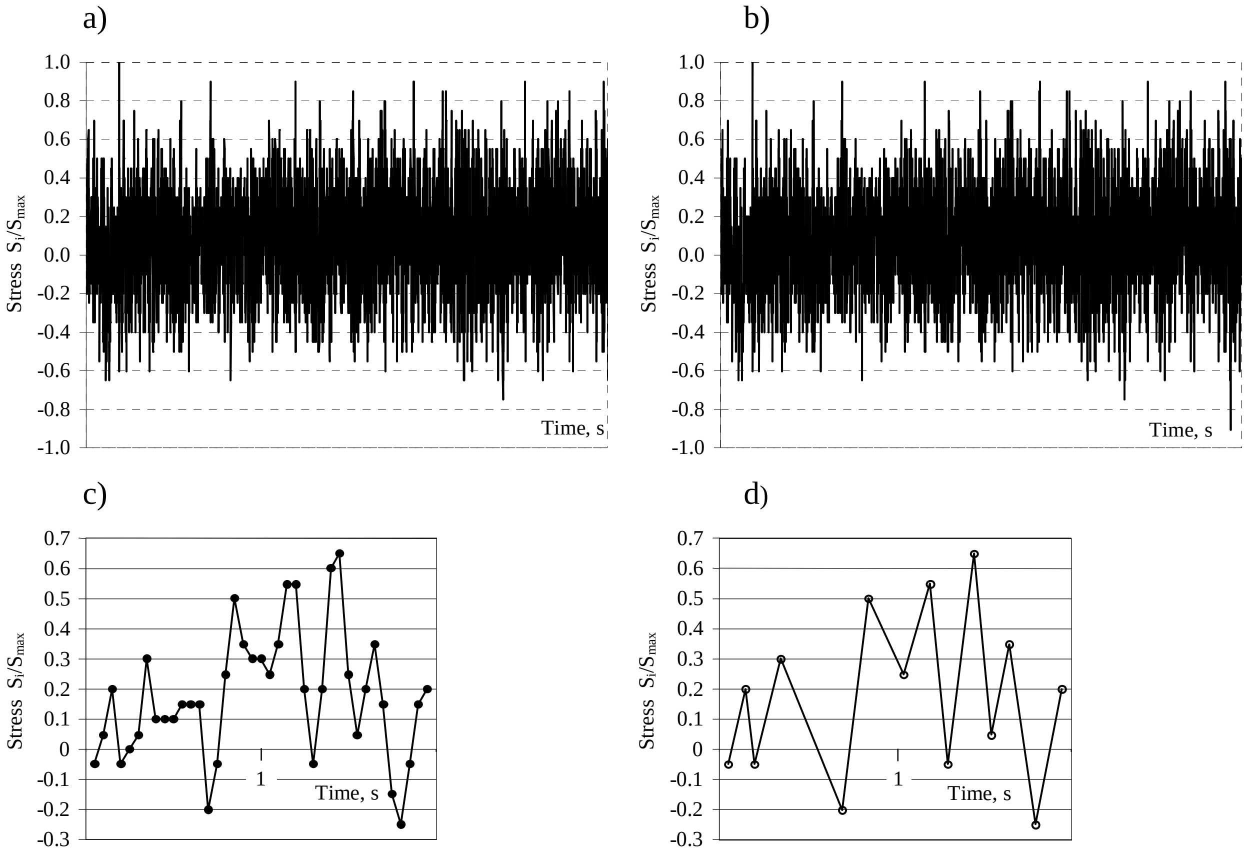

Fig. 1. Part of an exploitation course of loading for steering spindles registered during driving on concrete pavement: a – in discrete form, b – in form of local extrema, c – a short section of course depicted in the figure a, d – a short section of course depicted in the figure b

The forms of stress change course depicted above have been subjected to statistical evaluation, the aim of which was to determine: average value, variance, standard deviation, skewness, kurtosis and range. The results of the calculation have been depicted in the table 1.

Tab. 1. Statistical parameters of the loading course

Loading course in form	Statistical Parameters					
	Average Value	Variance	Standard Deviation	Skewness	Kurtosis	Range
Discrete	0.081	0.059	0.243	-0.0173	0.1825	1.97
Local extrema	0.076	0.089	0.298	0.0383	-0.4884	1.97
Difference between results δ, %	-6,2	50.8	22.6	321.4	-367,6	0.0

The calculated results differ from each other and difference values have been calculated from the relation (1)

$$\delta = \frac{L_0 - D_0}{D_0} \cdot 100\% \quad . \tag{1}$$

Cursory analysis of the examination results points at a small decline of the constituent statistical value of load in form of local extrema, which is described by the average value. In case of dynamical constituent for the course form mentioned above, described by a variance, the considerable increase 50,8% occurred. Significant differences concern the parameters characterizing the probability density function. In case of discrete form course the skewness value points at asymmetrical distribution of values regarding its average value. The asymmetry of the distribution extends in the direction of negative values. The value of kurtosis points at the relative culmination of the values distribution in comparison with normal distribution. The skewness and kurtosis values determined for the course in form of local extrema are considerably different from the values for course in discrete form. The value of the skewness coefficient points at an asymmetrical distribution extending in the direction of positive values, whereas on the base of kurtosis coefficient value it can be concluded that in comparison with normal distribution this distribution was relatively flat.

The differences ensuing in the statistical parameters for loading course in discrete form and form of local extrema should be explained by simplification connected with determination of values of extrema. The following part of the work will be based on the course in form of local extrema.

A significant property of the loading course, e.g. on account of the choice of the normalization method, is width of the load spectrum, which can be evaluated with use of the simplified method based on coefficient I value, expressed by the formula (2)

$$I = \frac{N_i}{N_e} . \tag{2}$$

The described method requires counting of local extrema, i.e. minimal and maximal values present in the course. Second element necessary to determine the coefficient I value is determination of the number of intersections of the average value level by ascending and diminishing high cycles. The average value for the course in form of local extrema has been depicted in table 1. As a result of calculations the number of intersections of average value level $N_i = 2115$ has been determined. The number of extrema in loading course totals $N_e = 2558$. By using the formula 2 the coefficient value $I = 0.827$ has been calculated. The value of the coefficient I enables us to qualify the analyzed course to wide-range loads. A detailed description of the method presented here can be found in reference [2].

3. Service load spectrum

The elaboration on service load spectrum requires so called normalization. In this work 4 methods have been applied: peak counting method (P.C.M), range counting method (R.C.M.), range pair count method (R.P.C.M.) and full cycles count method (F.C.C.M.). On the base of received set of sinusoidal cycles the bar charts depicting frequency of appearance of the cycles of determinate amplitude values S_{ai}/S_{max} and average value S_{mi}/S_{max} have been depicted (fig.2).

Tab. 2. Statement of statistical parameters for distribution of average values and amplitudes

Distribution	Method of normalization	Statistic parameters values			
		Average value	Standard deviation	Skewness	Kurtosis
$\frac{S_{ai}}{S_{max}}$	P.C.M.	0.303	0.163	0.6190	-0.2284
	R.C.M.	0.252	0.135	0.6190	0.0647
	R.P.C.M.	0.252	0.155	0.7715	0.1918
	F.C.C.M	0.252	0.154	0.7572	0.1305
$\frac{S_{mi}}{S_{max}}$	R.C.M.	0.132	0.140	-0.3750	0.7905
	R.P.C.M.	0.132	0.118	-0.5654	1.8912
	F.C.C.M	0.132	0.118	-0.5443	1.8779

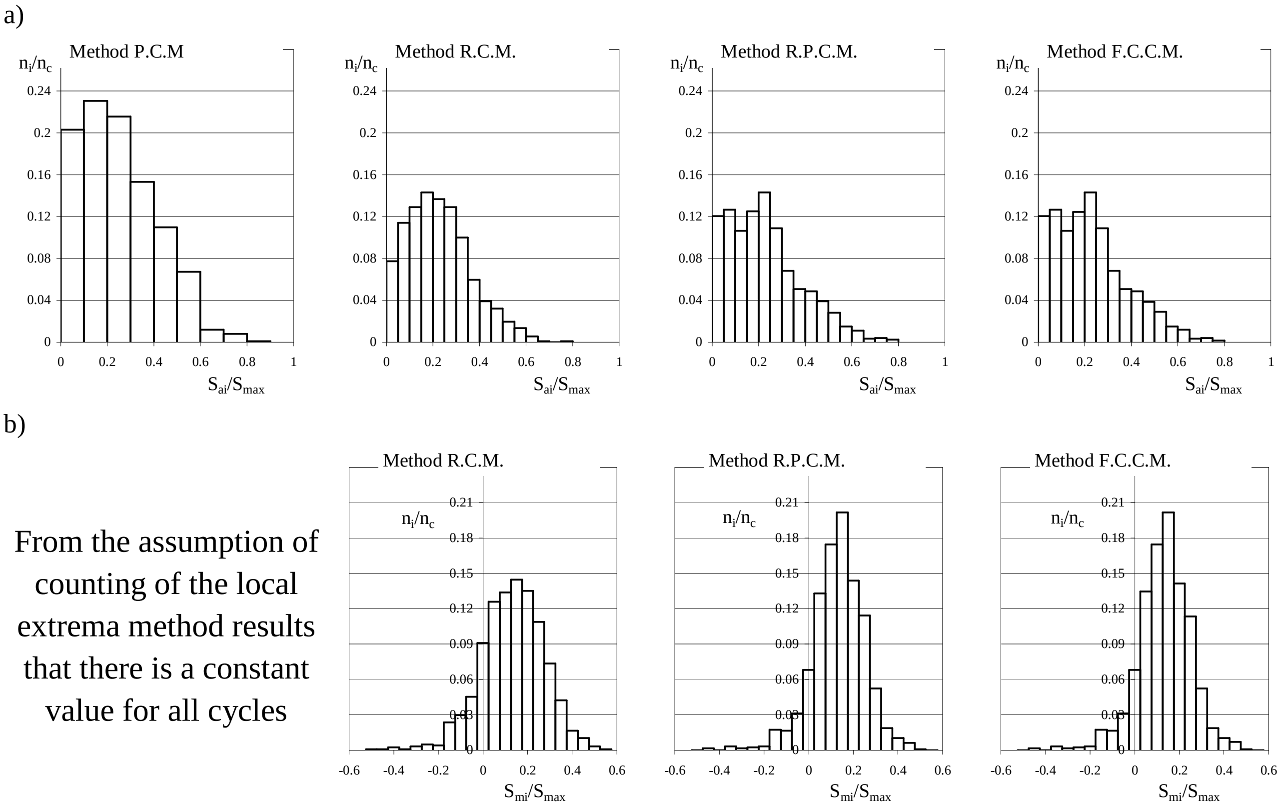

Fig. 2. Bar charts elaborated according to the data received as a result of normalization of service load: a – for amplitudes S_{ai}/S_{max}, b – for average value S_{mi}/S_{max}

The analysis of the distribution of average values and amplitudes can be carried out on the base of statistical parameters depicted in table 2. Superficial evaluation of data shows that the parameters of distribution of the average value for all methods of normalization are close to the values for normal distribution, whereas amplitude distribution is close to Raleigh distribution.

4. Two-parametric fatigue characteristics

The sinusoidal cycles determined as a result of normalization belonging to the service load have different amplitudes and average values, which has been signaled in superficial analysis in point above. On this account the calculations of fatigue life of structural component require assuming the characteristics which determines the fatigue properties for two variable parameters: amplitude and average value.

There are many well-known descriptions of fatigue surface [2, 7]. From the analysis of the fatigue characteristics described in the references mentioned it turns out that in case of limited number of data from fatigue research the good conformity between calculation and examination results can be received through the application of characteristics according to model II [2].

Model II consists in accepting a plane crossing the branch of limited durability of Haigh diagram determined with R = -1,0 and point R_m or R_e on the axis S_m as fatigue characteristics. This plane is described by a relation:

$$N = N_o[\frac{R_{-1}}{S_a}(1-\frac{S_m}{R_m})]^{m_o} \qquad \text{for} \qquad S_{max} \geq Z_G \,. \qquad (3)$$

5. Results of calculations and their analysis

The calculations of fatigue life have been carried out for S355J0 steel. On the base of the empirical examinations the properties in cyclic load and static load conditions, enabling us to elaborate on two-parametric fatigue characteristics, have been determined.

The strength properties of S355J0 steel received from the test of static tensile test are following: the yield point R_{eL} = 499.9 MPa (8.4 MPa), material tensile strength R_m = 678 MPa (7.1 MPa) and Young module E = 208159 MPa (1306 MPa). The standard deviation values for mentioned parameters have been given in brackets.

On the base of examination in constant amplitude load the S-N curve has been determined. The range of limited fatigue strength for oscilating load R = -1 is described by a relation:

$$\log S_{max} = -0.0811 \log N + 2.9247 . \tag{4}$$

On the base of slope in formula (4) the value of index exponent m_0 = 12,33 has been determined. The fatigue limit of the material totals R_{-1} = 274 MPa for number of cycles $N_0 = 10^6$.

In calculations the Palmgren-Miner linear damage hypothesis has been used. The total fatigue failure caused by all the cycles of variable values S_a and S_m is described by the formula (5)

$$D = \sum_{i=1}^{k} \sum_{j=1}^{p} D_{ij} = \sum_{i=1}^{k} \sum_{j=1}^{p} \frac{n_{ij}}{N_{ij}} . \tag{5}$$

Fatigue life in number of cycles is calculated from:

$$N_c = \frac{1}{D} \sum_{i=1}^{k} \sum_{j=1}^{k} n_{ij} . \tag{6}$$

After replacing the formula (3) with the data the equation describing two-parametric fatigue characteristics has been received:

$$N = 10^6 \left[\frac{274}{S_a} \left(1 - \frac{S_m}{678} \right) \right]^{12.33} . \tag{7}$$

As a result of the calculations with application of two-parametric fatigue characteristics (7), Palmgren-Miner hypothesis (5) and loading spectra the diagrams of fatigue life for accepted normalization methods (fig. 3) have been elaborated. The received results have been depicted against the background of fatigue life in service load conditions.

Analyzing the received calculation results some divergences have been noticed. The lowest strength have been received for loading spectrum elaborated with the peak counting method, whereas the biggest for the range counting method. The strength results received for the loading spectrum elaborated with the range pair count method and the full cycles count method are close to empirical examination results in range from 400 to 550 MPa.

In order to determine the differences in calculated fatigue life the quotient of analytical durability and durability in service load conditions has been determined for chosen maximal stress intensity in spectrum (fig.4).

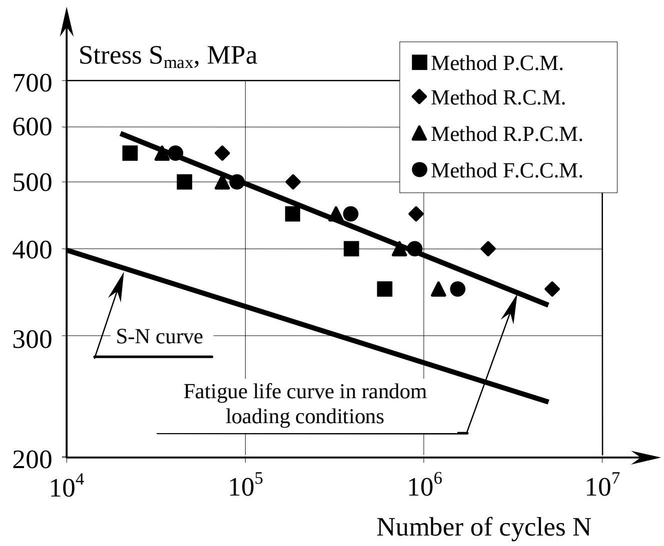

Fig. 3. The results of calculations of fatigue life

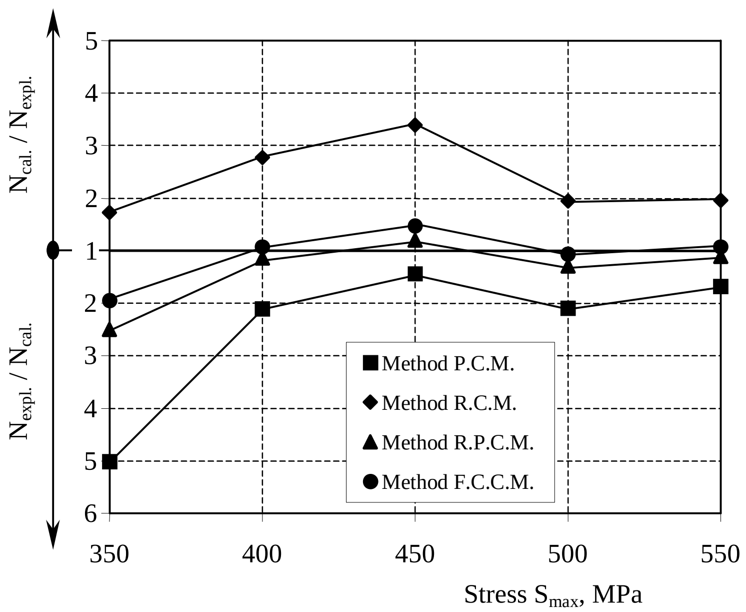

Fig. 4. Comparison of the durability results

Analyzing the examination results, shown in figure 4, good conformity between the calculation results and empirical examination results for the full cycles count method in the stress range from 400 to 550 MPa can be noticed. In the stress range mentioned above 1,5 growth of the analytical durability in comparison with experimental durability for value S_{max} = 450 MPa has been noted down. In case of the lowest stress value (350 MPa) the double decrease of the analytical value has been noted down. The similar tendency of the distribution of the results has been received for the load spectrum elaborated with the range pair count method. The scope of variability of analytical durability, for stresses from S_{max} = 400 MPa to S_{max} = 550 MPa, did not cross the level of 1.2. Similarly to the full cycles count method the biggest result divergence has been noted town on the level of 350 MPa, which amounted to approximately 2.5. The results of calculations for load spectrums elaborated with the range pair count method and the full cycles count method. were closest to experimental results for the range S_{max} = 400÷550 MPa.

In case of the peak counting method the calculation results are lower than empirical examination results for each stress intensivity. The scope of difference change totals from 1.4 (for stresses S_{max} = 450 MPa) to 5.0 (for stresses S_{max} = 350 MPa). Ensuing differences are related with the load spectrum, the shape of which results in the hardest loading conditions.

For the spectrum elaborated with the range counting method the results were higher from the results of empirical examination data. The differences oscillated within 1.7 and 3.4.

6. Summary

The comparison between the calculation results with the empirical examination results has pointed out the impact of the normalization method on the fatigue life. On the base of the received results the biggest conformity has been achieved for the range pair count method and the full cycles count method. The results achieved for the load spectrum elaborated with the full cycles count method are the closest to the empirical examination results, but depending on the chosen stress intensity the are located on the left and right side (in the safe and hazardous area) of the life fatigue diagram for random load. In case of the results for the range pair count method the durability results are located in the safe area with the exception of the S_{max} = 450 MPa. The close conformity of the results for the mentioned normalization methods is caused by a similar way of functioning. The differences in the sets of cycles have been visualized by small changes of the values of statistical parameters determined for distributions of average values and amplitudes. In case of the peak counting method, as expected, the lowest durability has been received, whereas in case of the range counting method the highest durability has been received.

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