OPTIMAL CONTROL ALGORITHMS FOR TRANSMISSIONS IN CITY BUSES

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Abstract

The article presents a mathematical model of motion of a car equipped with the automatic hydromechanical transmission. All possible conditions of transmission operation during car acceleration have been analysed. The model has been applied in a computer program for city bus acceleration simulation. Also the methodology of defining optimal gear shift algorithms for the automatic hydro mechanical transmission has been presented. There have been used two optimization criteria: acceleration time minimization and fuel consumption minimization in the acceleration phase. The methodology has been illustrated by an example of optimal control algorithms synthesis of the automatic hydromechanical transmission in a city bus.

Keywords: city bus, automatic hydromechanical transmission, control algorithms, optimization criteria, acceleration time, fuel consumption

1. Introduction

Nowadays production of buses and other automotive vehicles is very often based on compiling a complete product of sub-assemblies and parts supplied by other automotive companies specialized in a given field. Even small production plants undertake the task of building buses and use sub-assemblies and parts from recognized automotive business companies.

Such a strategy of designing and producing buses tailored to meet an individual client’s requests and using sub-assemblies and parts from famous automotive companies is applied by Polish bus and coach manufacturer Solaris Bus & Coach S.A. [6]. Their offer of a wide range of bus models for various purposes and with different equipment is a technological and commercial success. Buses from Solaris & Coach S.A. can be seen on the roads in Poland, Western and Central-Western Europe and the Middle East. They are mainly city buses.

2. City bus gearboxes

Public transport buses are continually improved to make them more comfortable for passengers, safer for the traffic, less burdensome for their drivers and more environmentally friendly. Because of all these aspects it is vital to replace bus manual mechanical drive system with an automatic system as well as continue work on improving the bus body design and modernizing its engine.
Automatic gearboxes are standard equipment of currently produced city buses [7]. They are automatic hydromechanical transmissions which bring many advantages to them.

The only important disadvantage of automatic hydromechanical transmissions applied in bus drive systems has so far been their less efficient exploitation compared to those with mechanical gearboxes and resulting a few per cent higher fuel consumption.

At present this essential drawback is successfully eliminated by means of:
– improving the transmission design with the aim of increasing its mean exploitation efficiency by optimization of the hydrokinetic transmission, the possibility of blocking it at top gears and the possibility of two-stream torque transmission at medium gears,
– optimizing transmission control programmes which realize assigned criteria of bus (car) motion quality with self-adaptation to variable exploitation factors.

These are also current tendencies in the development of automatic hydromechanical transmissions in world-wide known automotive companies with three companies dominating in the field of bus transmissions: Allison (USA), Zahnradfabrik Friedrichshafen AG (ZF) and Voith (Germany) [4, 5, 7].

3. Motion modelling for a bus with an automatic hydromechanical transmission

Bus drive system made up of ready-made sub-assemblies i.e. the engine, the gearbox, the drive shaft, the driving axle, axle shafts, road wheels, retarders etc. requires that they not only match mechanically and are properly situated in the body or chassis of a bus but also that their technical parameters and functional characteristics are chosen correctly and programs controlling cooperation of sub-assemblies prepared. This is especially true about algorithms of shifting gears control in a hydromechanic transmission in relation to the engine control and other bus parameters such as: mass, gearbox and main gear ratio, road wheels parameters, air resistance, road resistance and inertia resistance etc. There is a large dispersion of gear shift moments in transmission systems of automotive vehicles with manual mechanical gearboxes especially under city traffic conditions [2].

Incorrect algorithms for hydromechanical transmission gear shift control may lead to such undesired phenomena as:
– bus combustion engine operating at ranges undesired with respect to fuel consumption, dynamic properties or fumes toxicity,
– considerable variations in bus acceleration at adjacent gears which negatively influence passengers’ comfort and cause dynamic overload in the torque converting system which reduces drive system life.

In order to improve hydromechanical transmission design and optimize its control programs through computer simulations of bus motion it is necessary to employ mathematical models describing processes of the bus motion.

4. Research object characteristics

The studied object is a city bus Solaris Urbino 12 weighing 13200 kg (at the time of research) with a 9.2 dm³ compression-ignition supercharged engine DAF PR183 [6] of the power of 183 kW at 2200 rpm and the torque 1050 Nm at 1100 – 1700 rpm, equipped with an automatic hydromechanical transmission ZF 5HP500 [7] with the maximum transformation coefficient of 2.5 and a possibility of blocking the hydrokinetic transmission if its operation is not indispensable under given traffic conditions. This contributes to increasing average exploitation efficiency of the hydromechanical transmission. The kinematic diagram of the hydromechanical transmission is presented in fig. 1 and the sequence of elements shifting respective gears in tab. 1.
5. Acceleration model of a bus with the automatic hydromechanical transmission

In the process of car acceleration the power stream transmission in the hydromechanical transmission can be realized in the following way: as a one stream flow through the hydrokinetic transmission in the first, second (and reverse) gears and mechanically with a blocked hydrokinetic transmission in the third, fourth and fifth gears. In each of the above mentioned situations mechanical transmission gear shifts can be accomplished. The mathematical specification of the dynamics of the equivalent model of a car drive with a hydromechanical transmission will also change accordingly.

Following d’Alembert’s principle and skipping transitional transformations we get the following mathematical dependences which represent the bus acceleration model for respective operation conditions of the hydromechanical transmission [1]:

– Monotonic acceleration with full-stream power transmission through the hydrokinetic transmission in between gear shifts:

\[
\begin{align*}
M_S - M_p &= I_S \frac{d\omega_S}{dt} \\
\left( F_n - \eta mg - c_x \frac{\gamma A}{2} v^2 \right) i_0 i_i \frac{r_d}{r_d} &= \left[ m + \sum \frac{I_k}{r_d^2} + I_m \left( \frac{\eta_m i_i}{r_d} \right)^2 \right] \frac{d\omega_T}{dt} \\
i_d M_p \frac{\eta_m i_i}{r_d} - \eta mg - c_x \frac{\gamma A}{2} v^2 &= \left[ m + \left( i_d I_S \frac{d\omega_S}{dt} + I_m \left( \frac{\eta_m i_i}{r_d} \right)^2 \right) \right] \frac{dv}{dt} + \sum \frac{I_k}{r_d^2}
\end{align*}
\]
where:

\( M_S \) – engine torque,
\( M_p \) – output torque on hydraulic converter pump impeller,
\( I_S \) – moment of inertia of engine crankshaft together with the flywheel, pump impeller and the liquid connected to it,
\( I_{un} \) – moment of inertia of the masses of power transmission system elements between converter output shaft and driven road wheels,
\( I_k \) – moment of inertia of a single wheel together with rotating brake elements,
\( F_n \) – driving force,
\( \Psi \) – road resistance coefficient \( \Psi = f \cos \alpha + \sin \alpha \),
\( m \) – bus total weight,
\( g \) – gravitational acceleration,
\( e_x \) – air resistance coefficient,
\( \gamma \) – air density,
\( A \) – car side face,
\( v \) – car velocity,
\( r_d \) – wheel dynamic radius,
\( i_0 \) – final drive ratio,
\( i_i \) – ratio of shifted reduction gear,
\( \eta_m \) – mechanical efficiency of car power transmission system,
\( \omega_S \) – engine angular velocity,
\( \omega_T \) – angular velocity of hydraulic converter turbine wheel,
\( i_d \) – dynamic ratio of hydraulic torque converter (transformation coefficient).

– Monotonic acceleration in case of the blocked hydromechanical transmission is similar to the one for the car with a mechanical gearbox in respect of mathematical specification:

\[
M_S \frac{\eta_m i_0 i_i}{r_d} - \Psi m g - c_x \frac{\gamma A}{2} v^2 = \left[ m + \left( I_S + I_{un} \left( \frac{\eta_m i_0 i_i}{r_d} \right)^2 + \sum I_k \right) \right] \frac{dv}{dt}.
\]  

(2)

– Gear shifts during the full-stream torque transmission in the hydrokinetic transmission:

\[
\begin{align*}
M_S - M_p &= I_S \frac{d\omega_S}{dt} \\
i_d M_p - M_{ii-1} &= I_T \frac{d\omega_T}{dt} \\
\left( M_{ii} i_i^b + M_{ii-1} i_i-1^b \right) \frac{\eta_m i_0}{r_d} - \Psi m g - c_x \frac{\gamma A}{2} v^2 &= \left[ m + I_{un} \left( \frac{\eta_m i_0 i_i}{r_d} \right)^2 + \sum I_k \right] \frac{dv}{dt}
\end{align*}
\]

(3)

where:
\( M_{ii}, M_{ii-1} \) – friction torque on the clutches shifting \( i \)-th and \( i-1 \)-st gear,
\(i^c_i, i^c_{i-1}\) – ratios of transmission reduction gear in the sector between turbine wheel and active (attacking) clutch plates shifting \(i\)-th and \(i-1\)-st gear,

\(i^b_i, i^b_{i-1}\) – ratios of transmission reduction gear in the sector between passive (being attacked) clutch plates shifting \(i\)-th and \(i-1\)-st gear and converter output shaft,

\(I_T\) – moment of inertia of hydraulic torque converter turbine wheel and the liquid connected to it.

- The hydrokinetic transmission blockade at \(i\)-th gear:

\[
M_S - M_p - M_{bl} = I_S \frac{d\omega_S}{dt}
\]

\[
\left(\frac{r_d i_d M_p + M_{bl}}{\eta_m i_0 i_i} - \Psi mg - c_x \frac{v^2}{2}\right) = \left[ m + \left(\frac{i_d I_S}{d\omega_T} \right) \left(\frac{\eta_m i_0 i_i}{r_d}\right)^2 + \sum I_k \frac{r_d^2}{r_d^2} \right] \frac{dv}{dt},
\]

where:

\(M_{bl}\) – friction torque on the clutch blocking hydraulic torque converter.

- Gear shifting when the hydrokinetic transmission is blocked:

\[
M_S - \frac{M_{\dot{u}}}{i_i^c} - \frac{M_{\dot{u} - 1}}{i_{i-1}^c} = (I_S + I_{un}) \frac{d\omega_S}{dt}
\]

\[
\left(\frac{r_d M_{\dot{u}} i_i^b + M_{\dot{u} - 1} i_{i-1}^b}{\eta_m i_0} - \Psi mg - c_x \frac{v^2}{2}\right) = \left[ m + I_{un} \left(\frac{\eta_m i_0 i_i}{r_d}\right)^2 + \sum I_k \frac{r_d^2}{r_d^2} \right] \frac{dv}{dt}
\]

Dependences (1)-(5) were applied while the program for computer simulation of acceleration of a bus with an automatic hydromechanical transmission was worked out, then it was realized and initially verified experimentally in operation [3].

**6. Algorithms of shifting gears in the automatic hydromechanical transmission**

As a rule popular methods of defining gear shift algorithms for an automatic gearbox refer to car motion maximally similar to steady motion. Difficulties with direct application of car theory criteria like fuel consumption economy and acceleration dynamics but also applied imperfect mathematical methods have determined a variety of additional factors which show graphically or graphically and analytically optimal gear shifts moments. An assessment of these factors and the physical reason behind them can be found in this study [1].

First of all, recommended gear shifts which ensure the highest dynamics of acceleration should be performed when the engine reaches its rotational speed equal to its maximum power or they can be described as crossing points of engine power curves in the function of motion velocity at two adjacent gears. Moreover, drive power on driven wheels curves, dynamic and car acceleration characteristics and engine maximum rotational speed moments can also be applied as economical optimality factors of this gear shift strategy.

Gear shift moments which guarantee the least fuel consumption are defined as crossing points of hourly fuel consumption curves in the function of vehicle motion velocity at stable fuel intake or unitary fuel consumption curves at adjacent gears. Other factors in optimally economical car acceleration are unitary amount of fuel consumption with regard to hydrokinetic transmission efficiency or drive power on car wheels with equal fuel consumption per hour at adjacent gears of...
hydromechanical transmission or else hydrokinetic transmission efficiency in the function of motion velocity.

Such variety of applied factors and therefore many different methods of setting gear shift optimal moments obviously leads to vital differences between gear shift moments algorithms defined for the same hydromechanical transmission. This is a result of different approximation of the above mentioned factors in relation to initial optimization criteria and this contributes to significant divergences between hydromechanical transmission algorithms based on the same optimization criteria. Hence the need arises to assess them from the point of view of ensuring required results extremum and to see to what extent the applied factors meet the original optimization criteria.

7. Optimization criteria and acceleration quality functionals

Determining optimal gear shift moments in a hydro-mechanical transmission at constant position of the throttle pedal means finding such car motion velocity values \( V_p \), at which gear shifts should be done – shifts from lower to higher gears. This should ensure obtaining functional extremum of the quality of car acceleration. Because of the well-known diversity between acceleration dynamics and fuel economy it seems justified to employ a few criteria simultaneously. This allows them to complement each other and makes control of the obtained results possible. Analysis of similar studies indicates that the following initial criteria of car acceleration optimization should be adopted:

- acceleration time \( T \) necessary to reach the assigned terminal acceleration velocity \( V_k \),
- distance covered \( S \) at accelerating up to assigned terminal velocity \( V_k \),
- fuel consumption \( Q \) necessary to reach the assigned terminal acceleration velocity \( V_k \),
- variational criterion \( \varepsilon \) of fuel consumption at accelerating with acceleration dynamics taken into account [1].

Appropriate bus acceleration quality functionals for argument \( v \) can be presented as [1]:

- for acceleration dynamics:

\[
J_T = \sum_{i=1}^{n} \int_{V_0}^{V_i} \frac{1}{a_i(v)} \, dv = \sum_{i=1}^{n} \int_{V_0}^{V_i} \frac{1}{a_i(v)} \, dv + \int_{V_p}^{V_k} \frac{1}{a_{i+1}(v)} \, dv \rightarrow \min ,
\]

\[
J_S = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} \left[ \int_{V_0}^{V_i} \frac{v}{a_i(v)} \, dv + \int_{V_p}^{V_k} \frac{v}{a_{i+1}(v)} \, dv - V_k \left( \int_{V_0}^{V_i} \frac{1}{a_i(v)} \, dv + \int_{V_p}^{V_k} \frac{1}{a_{i+1}(v)} \, dv \right) \right] \rightarrow \min ,
\]

- for fuel consumption:

\[
J_Q = \sum_{i=1}^{n} \int_{V_0}^{V_i} \frac{(g_e N_e)}{a_i(v)} \, dv = \sum_{i=1}^{n} \int_{V_0}^{V_i} \frac{(g_e N_e)}{a_i(v)} \, dv + \int_{V_p}^{V_k} \frac{(g_e N_e)}{a_{i+1}(v)} \, dv \rightarrow \min ,
\]
\[
J_e = \sum_{i=1}^{n} Q_i - \sum_{i=1}^{n} \left( \frac{g_e N_e}{v_k} \right) S_i = \sum_{i=1}^{n} \left[ \int_{v_0}^{v_p} \frac{\left( \frac{g_e N_e}{a_i(v)} \right)}{v_k} dv + \int_{v_p}^{v_k} \frac{\left( \frac{g_e N_e}{a_{i+1}(v)} \right)}{v_k} dv \right] - \left( \frac{g_e N_e}{v_k} \right)_k \left[ \int_{v_0}^{v_p} \frac{v}{a_i(v)} dv + \int_{v_p}^{v_k} \frac{v}{a_{i+1}(v)} dv \right] \to \min,
\]

where:
- \(a\) – bus acceleration,
- \(v\) – motion velocity actual value,
- \(V_0, V_k\) – initial and terminal acceleration velocity,
- \(i, n\) – index and number of gears in hydromechanical transmission, respectively,
- \(S_i, t_i\) – distance covered and acceleration time at \(i\)-th shift gear, respectively,
- \(g_e\) – specific fuel consumption of engine,
- \(N_e\) – engine power output.

In order to analytically investigate extremum of expressions (6)-(9) the integrand was expressed in bus motion velocity function \(v\) by design parameters and functional dependences characteristic of the studied object.

A mathematical model of car acceleration described by dependences (1)-(5) and an assumption that bus motion is a particle motion has been used to define optimal gear shift moments of the hydromechanical transmission at constant throttle pedal position. This can be expressed as follows:

\[
a = \frac{F_n - F_{op}}{m_{red}},
\]

where:
- \(F_{op}\) – motion resistance force,
- \(m_{red}\) – bus reduced mass including inertia in the acceleration process.

Reduced mass of a car with hydrokinetic transmission in the drive system is defined as:

\[
m_{red} = m\delta = m\left\{ 1 + \frac{1}{m r_d^2} \left[ \left( I_p I_d \frac{d\omega_p}{d\omega_T} + I_T \right) (i_g i_t)^2 \eta_m + \sum I_k \right] \right\},
\]

where:
- \(\delta\) – reduced mass coefficient,
- \(I_p\) – inertia moment of a hydrokinetic transmission pump impeller and engine rotating elements,
- \(d\omega_p, d\omega_T\) – angular acceleration of pump and hydrokinetic transmission turbine impellers, respectively.

With the use of prepared mathematical model of bus acceleration (of a bus with hydromechanical transmission) and determined motion quality functionals the conditions of their extremum existence were defined by computer simulations. Optimal gear shifts of the hydromechanical transmission should take place at the moment when the accelerating vehicle reaches such velocity \(V_p\) at which the following conditions are fulfilled:
for acceleration time criterion $T$ to assigned velocity $V_k$:

$$
\frac{d}{dv} \left[ \int_{V_0}^{V_p} \frac{1}{a_i(v)} dv - \int_{V_p}^{V_k} \frac{1}{a_{i+1}(v)} dv \right] = \frac{1}{a_i(V_p)} dv - \frac{1}{a_{i+1}(V_p)} dv = 0, \quad (12)
$$

for acceleration distance covered criterion $S$ to assigned velocity $V_k$:

$$
\frac{d}{dv} \left[ \int_{V_0}^{V_p} a_i(v) dv - \int_{V_p}^{V_k} a_{i+1}(v) dv - V_k \left[ \int_{V_0}^{V_p} \frac{1}{a_i(v)} dv - \int_{V_p}^{V_k} \frac{1}{a_{i+1}(v)} dv \right] \right] =
$$

$$
= \frac{V_p}{a_i(V_p)} - \frac{V_p}{a_{i+1}(V_p)} - V_k \left[ \frac{1}{a_i(V_p)} - \frac{1}{a_{i+1}(V_p)} \right] = 0, \quad (13)
$$

for fuel consumption $Q$:

$$
\frac{d}{dv} \left[ \int_{V_0}^{V_p} (g_eN_e)_i a_i(v) dv - \int_{V_p}^{V_k} (g_eN_e)_{i+1} a_{i+1}(v) dv \right] =
$$

$$
= \frac{(g_eN_e)_i}{a_i(V_p)} - \frac{(g_eN_e)_{i+1}}{a_{i+1}(V_p)} \left[ \frac{V_p}{a_i(V_p)} - \frac{V_p}{a_{i+1}(V_p)} - V_k \left[ \frac{1}{a_i(V_p)} - \frac{1}{a_{i+1}(V_p)} \right] \right] = 0, \quad (14)
$$

for variational fuel consumption $\mathcal{E}$ :

$$
\frac{d}{dv} \left[ \int_{V_0}^{V_p} (g_eN_e)_{i+1} \left( (g_eN_e)_i \right) a_i(v) dv - \int_{V_p}^{V_k} (g_eN_e)_{i+1} a_{i+1}(v) dv \right] =
$$

$$
= \left[ \frac{V_p}{a_i(V_p)} - \frac{V_p}{a_{i+1}(V_p)} - V_k \left[ \frac{1}{a_i(V_p)} - \frac{1}{a_{i+1}(V_p)} \right] \right] = 0. \quad (15)
$$

Conditions (12) and (13) concerning acceleration dynamics can be expressed as:

$$
[a_i(V_p) - a_{i+1}(V_p)] [V_p - V_k] = 0. \quad (16)
$$

Similarly gear shift optimality conditions with regard to fuel consumption can be formulated as:

$$
\left[ \frac{(g_eN_e)_i}{a_i(V_p)} - \frac{(g_eN_e)_{i+1}}{a_{i+1}(V_p)} \right] \left[ \frac{V_p}{a_i(V_p)} - \frac{V_p}{a_{i+1}(V_p)} - V_k \left[ \frac{1}{a_i(V_p)} - \frac{1}{a_{i+1}(V_p)} \right] \right] = 0. \quad (17)
$$

Optimal gear shift moments in the hydromechanical transmission for a bus acceleration process can be received by finding the extremum of the above presented functionals. According to dynamics criteria they will be defined as crossing points of the vehicle acceleration curves in the function of motion velocity at adjacent gears. If there are no such crossing points, gear shift moments will be defined as limit points of the interval of possible velocity variability $V_p$ at the preceding gear.
This procedure strategy is illustrated in fig. 2 where acceleration curves at respective gears of the Solaris bus equipped with an automatic hydromechanical transmission ZF 5HP500 have been shown. The crossing points of these accelerations mark mechanical reducer gear shift moments which ensure maximum bus acceleration dynamics.

According to optimal economy criterion moments of gear shifts are defined by the crossing points of factors curves expressed as: \( \frac{g_e N_e}{a} \) in the function of the vehicle motion velocity at adjacent gears in the hydromechanical transmission or as border range of possible velocity variability interval \( V_p \). Fig. 3 shows graphically how optimal gear shift moments are determined so that they minimize fuel consumption of the Solaris bus equipped with automatic transmission ZF 5HP500 and yet required acceleration dynamics is preserved.

Optimal algorithms for gear shift control in the hydromechanical transmission ZF 5HP500 of the Solaris city bus based on such strategies have been graphically illustrated in fig. 4 as gear shift lines for the bus acceleration phase. Both algorithms realizing maximum bus acceleration
dynamics and those ensuring minimisation of fuel consumption in the process of acceleration have
been distinguished.

Fig. 4. Algorithms of automatic hydromechanical transmission ZF 5HP500 control in the Solaris city bus

8. Conclusions

According to various criteria of analyzing acceleration dynamics or fuel consumption
conditions of optimal gear shift moments are the only ones and they are not contradictory in terms
of quality. Employing various criteria leads only to diversifying quantity assessment of motion
dynamics or fuel consumption in the same bus acceleration process under study.

Undertaken analytical and experimental [1, 3] studies applied to automatic hydromechanical
transmissions have confirmed optimality of gear shift moments defined by this method. A set of
such moments in the whole area of engine power control defines gear shift algorithms in the
hydromechanical transmission and they in turn define the control programs. The hydromechanical
transmission control programs have a considerable impact on fuel-traction properties of the bus
cyclical motion and thus on technical and economical effectiveness of its exploitation.

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